Tetrad Gravity and Dirac's Observables.

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In a recent series of papers [1] the canonical reduction of a new formulation of tetrad gravity to a canonical basis of Dirac's observables in the 3-orthogonal gauge in the framework of Dirac-Bergmann theory of constraints[2] was studied.

This concludes the preliminary work in the research program aiming to give a unified description of the four interactions in terms of Dirac's observables. See Ref.[3] for a complete review of the achievements obtained till now:
i) The understanding of the mathematical structures involved, in particular of the Shanmugadhasan canonical transformations [4, 5].
ii) The non-manifestly covariant canonical reduction to a canonical basis of Dirac's observables of many relativistic systems, including relativistic particles, the Nambu string, the electromagnetic, Yang-Mills and Dirac fields, the standard SU(3)xSU(2) xU(1) model of elementary particles in Minkowski spacetime. In the case of gauge theories, this required an understanding of all the pathologies of the constraint manifold associated with the Gribov ambiguity (gauge symmetries and gauge copies) and of the fact that the presence or absence of the Gribov ambiguity depends on the choice of the function space for the gauge fields and the gauge transformations. With the hypothesis that no physics is hidden in the Gribov ambiguity, on can work in special weighted Sobolev spaces [6] where it is absent. Then, in the case of trivial principal bundles on constant time hypersurfaces in Minkowski spacetime (no monopoles; winding number but not instanton number) and for suitable Hamiltonian boundary conditions on gauge potentials and gauge transformations (the behaviour at spatial infinity must be direction-independent) allowing the color charges, in the

case of SU(3), to be well defined, one can do a complete canonical reduction of Yang-Mills theory like in the electromagnetic case and find the singularity-free physical Hamiltonian.
iii) The definition of the Wigner-covariant rest-frame instant form of dynamics (replacing the non-relativistic separation of the center-of-mass motion) for the timelike configurations of every isolated relativistic system (particles, strings, fields) in Minkowski spacetime. This is obtained starting from the reformulation of the isolated system on arbitrary spacelike hypersurfaces (parametrized Minkowski theories) and making a restriction to the special foliation (3+1 splitting) of Minkowski spacetime with Wigner hyperplanes: they are determined by the given configuration of the isolated system, being orthogonal to its conserved total 4-momentum (when it is timelike). A general study of the relativistic center of mass, of the rotational kinematics and of Dixon multipolar expansions [8] is now under investigation [9] for N-body systems. See Refs.[10, 11] for the center of mass of a Klein-Gordon configuration.
iv) The Wigner-covariant reformulation of the previous canonical reductions in the rest-frame instant form taking into account the stratification of the constraint manifold associated with the isolated system induced by the classification of its configurations according to the Poincaré orbits for the total 4-momentum.
v) The realization that in the rest-frame instant form there is a universal breaking of Lorentz covariance regarding only the decoupled canonical non-covariant "external" center of mass (the classical analogue of the Newton-Wigner position operator), while all the relative degrees of freedom are Wigner-covariant. The spacetime spreading of this non-covariance determines a classical unit of length, the Møller radius[12], which is determined by the value of the Poincaré Casimirs of the given configuration of the isolated system and which should be used as a physical ultraviolet cutoff in quantization. The Møller radius is a non-local effect of Lorentz signature: already at the classical level it is impossible to localize in a covariant way the canonical center of mass of an isolated extended relativistic system with a precision better of this radius. This classical problem happens at those distances where quantum mechanics introduces pair creation (the Møller radius is of the order of the Compton wavelength of the isolated system). Moreover, the Møller radius is a remnant in Minkowski spacetime of the energy conditions of general relativity. With the methods of Ref.[13] one can find the "internal" 3-center of mass inside the Wigner hyperplane, whose vanishing is the gauge fixing for the constraints defining the rest frame.
vi) Since the rest-frame instant form is a special classical background for the Tomonaga-Schwinger formulation of quantum field theory, there is now the possibility to start with a Wigner-covariant quantization of field theory on Wigner hyperplanes. Having built-in a covariant concept of "equal time", one expects to find a Schroedinger-like equation for relativistic bound states (avoiding the problem of the spurious solutions of the Bethe-Salpeter equation), to be able to define Tomonaga-Schwinger asymptotic states (with the possibility of including bound states among them) and to use

the Møller radius as a physical ultraviolet cutoff.

The next conceptual problem was to apply all the technology developed for constrained systems in Minkowski spacetime to a formulation of general relativity able to incorporate the standard model of elementary particles and such that it could be possible to formulate a deparametrization scheme according to which the switching off of the Newton constant reproduces the description of the standard model on the Wigner hyperplanes in Minkowski spacetime. In this way, at least at the classical level, the four interactions would be described in a unified way and one could begin to think how to make their quantization in a way avoiding the existing incompatibilty between quantum mechanics and general relativity.

Tetrad gravity, rather than metric gravity, was the natural formulation to be used for achieving this task for the following reasons:
i) The fermions of the standard model couple naturally to tetrad gravity.
ii) Tetrad gravity incorporates by definition the possibility to have the matter described by an arbitrary (geodesic or non-geodesic) timelike congruence of observers. In this way one can arrive at a Hamiltonian treatment of the precessional aspects of gravitomagnetism like the Lense-Thirring effect[14].
iii) In tetrad gravity it is possible to replace the supermomentum constraints with SO(3) Yang-Mills Gauss laws associated with the spin connection and to solve them with the technology developed for the canonical reduction of Yang-Mills theories. Instead in metric gravity one does not know how to solve the supermomentum constraints.

Let us remark that till now supergravity and string theories have not been analyzed, since the emphasis is on learning how to make the canonical reduction in presence of constraints and from this point of view these theories only have bigger gauge groups and many more constraints to be solved.

Another important point is that the dominant role of the Poincaré group and of its representations in the theory of elementary particles in Minkowski spacetime requires to formulate general relativity on non-compact spacetimes asymptotically flat at spatial infinity so that the asymptotic Poincaré charges [15, 16] exist and are well defined. In presence of matter these asymptotic Poincaré charges must reproduce the ten conserved Poincaré generators of the isolated system with same matter content when the Newton constant is switched off.

All these requirements select a class of spacetimes with the following properties:
i) They are pseudo-Riemannian globally hyperbolic 4-manifolds ($M^4 \approx R \times \Sigma, {}^4g$) $[(\tau, \vec{\sigma}) \mapsto z^\mu(\tau, \vec{\sigma})]$. These spacetimes have a global time function $\tau(z)$ and admit 3+1 splittings corresponding to foliations with spacelike hypersurfaces Σ_τ (simultaneity 3-manifolds, which are also Cauchy surfaces).
ii) They are non-compact and asymptotically flat at spatial infinity.
iii) They are parallelizable 4-manifolds, namely they admit a spinor structure and have trivial orthonormal frame principal SO(3)-bundles over each simultaneity 3-manifold Σ_τ.
iv) The non-compact parallelizable simultaneity 3-manifolds Σ_τ are assumed to be

topologically trivial, geodesically complete and diffeomorphic to R^3 [$\Sigma_\tau \approx R^3$]. This implies the existence of global coordinate systems on Σ_τ, so that coordinate systems $(\tau, \vec{\sigma})$, adapted to the simultaneity 3-surfaces Σ_τ, can be used for M^4. In this simplified case the geodesic exponential map is a diffeomorphism, there are no closed 3-geodesics and no conjugate Jacobi points on 3-geodesics.
v) The cotriads on Σ_τ and the associated 3-spin-connection on the orthogonal frame SO(3)-bundle over Σ_τ are assumed to belong to suitable weighted Sobolev spaces so that the Gribov ambiguity is absent. This implies the absence of isometries (and of the associated Killing vectors) of the non-compact Riemannian 3-manifold $(\Sigma_\tau, {}^3g)$.
vi) Diffeomorphisms on Σ_τ and their extension to tensors are interpreted in the passive sense (pseudo-diffeomorphisms), following Ref.[17], in accord with the Hamiltonian point of view that infinitesimal diffeomorphisms on tensors are generated by taking the Poisson bracket with the first class supermomentum constraints.

As action principle we use the ADM metric action with the 4-metric ${}^4g$ rewritten in terms of general cotetrads on M^4. For the general cotetrads a new special parametrization has been found. Starting from Σ_τ-adapted cotetrads (Schwinger time gauge) whose 13 degrees of freedom are the lapse and shift functions and the cotriads on Σ_τ, the remaining 3 degrees of freedom are described by the 3 parameters which parametrize timelike Wigner boosts acting on the flat indices of the cotetrad (in the cotangent spaces over each point of Σ_τ). This implies that the flat indices acquire Wigner covariance (the time index becomes a Lorentz scalar, while the spatial indices become Wigner spin 1 3-indices) in each point of Σ_τ. These 3 boost parameters describe the transition from the Σ_τ-adapted Eulerian observers associated with the Σ_τ-adapted tetrads (this timelike congruence is surface-forming and is orthogonal to the Σ_τ's) to an arbitrary (in general not surface-forming) timelike congruence of observers.

The ADM Lagrangian density is considered a function of these 16 fields: the lapse and shift functions, the cotriads on Σ_τ, the 3 boost parameters. In tetrad gravity there are 14 first class constraints (10 primary and 4 secondary ones):
i) The momenta conjugate to the lapse and shift functions vanish, so that lapse and shifts are 4 arbitrary gauge variables [arbitrariness in the choice of the standard of proper time and conventionality in the choice of the notion of simultaneity with the associated possible anisotropy in light propagation].
ii) The momenta conjugate to the 3 boost parameters vanish (Abelianization of the Lorentz boost contained in the SO(3,1) group acting on the flat indices of the cotetrads): the 3 boost parameters are arbitrary gauge variables (the physics does not depend on the choice of the timelike congruence of observers).
iii) There are 3 constraints describing the generators of SO(3) rotations on the flat indices of the cotetrads: the associated 3 angles (3 degrees of freedom among the 9 parametrizing the cotriads) are gauge variables (conventionality in the choice of the standard of non-rotation for a timelike congruence of observers).
iv) There are 3 secondary constraints which are equivalent to the ADM supermomentum ones (it is possible to replace them with SO(3) Yang-Mills Gauss laws for

the spin connection): 3 degrees of freedom, depending on the cotriads and their time derivatives, are arbitrary gauge variables describing the freedom in the choice of the 3-coordinates on Σ_τ (arbitrariness in the choice of 3 standards of length). These 3 constraints generate the pseudo-diffeomorphisms.
v) One secondary constraint coincides with the ADM superhamiltonian one. It can be shown that this constraint has to be interpreted as the Lichnerowicz equation [18]determining the conformal factor of the 3-metric ${}^3g$. Therefore, the last gauge variable is the momentum conjugate to this conformal factor [it is non-locally connected with the trace of the extrinsic curvature of Σ_τ, also named York time [19]] and the gauge transformations generated by the superhamiltonian constraint correspond to the transition from one allowed 3+1 splitting of M^4 with spacelike hypersurfaces Σ_τ to another one (the physics does not depend on the choice of the 3+1 splitting, like in parametrized Minkowski theories).

In conclusion, there are only two dynamical degrees of freedom hidden in the cotriads on Σ_τ and they describe the gravitational field. Their determination requires a complete breaking of general covariance, namely a complete fixation of the gauge degrees of freedom (this amounts to the choice of a physical laboratory where to do all the measurements).

Let us remark that the fixation of the 3-coordinates and of the 3 rotation angles are inter-related, because the associated constraints do not have vanishing Poisson brackets. Moreover, there are restrictions on the gauge transformations when one restricts himself to the solutions of Einstein's equations: according to the general theory of constraints one has to start by adding the gauge fixings to the secondary constraints; the requirement of their time constancy generates the gauge fixings of the primary constraints. Therefore, since the supermomentum constraints are secondary ones, the choice of the 3-coordinates on Σ_τ determines the choice of the shift functions (i.e. of the associated convention for simultaneity in M^4; the Einstein convention can be applied only when the shift functions vanish). Analogously, the choice of the 3+1 splitting of M^4 (fixation of the momentum conjugate to the conformal factor of the 3-metric) determines the choice of the lapse function (namely of how the 3-surfaces Σ_τ are packed in the chosen 3+1 splitting of M^4).

The next problem is the choice of the boundary conditions for the 16 fields in the cotetrads and for the allowed Hamiltonian gauge transformations. The existence of the Poisson brackets and the differentiability of the Dirac Hamiltonian require the addition of a surface term [20] to the Dirac Hamiltonian containing the strong ADM Poincaré charges: they are surface integrals at spatial infinity, which differ from the weak ADM Poincaré charges (volume integrals) by terms vanishing due to the secondary constraints. In spacetimes asymptotically flat at spatial infinity besides the 10 asymptotic Poincaré charges there is a double infinity of Abelian supertranslations (associated with the asymptotic direction-dependent symmetries of these spacetimes [21]). Their presence generates an infinite-dimensional algebra of asymptotic charges which contains an infinite number of conjugate Poincaré subalgebras: this forbids the identification of a well defined angular momentum in general

relativity. The requirement of absence of supertranslations, so to have a uniquely defined asymptotic Poincaré algebra, puts severe restrictions on the boundary conditions of the 16 fields and of the gauge transformations.

Following Dirac [2], we assume the existence of asymptotic flat coordinates for M^4. It can be shown that this implies the restriction of the allowed 3+1 splittings of M^4 to those whose associated foliations have the leaves Σ_τ approaching spacelike Minkowski hyperplanes at spatial infinity. The absence of supertranslations requires that this approach must happen in a direction-independent way and that the lapse and shift functions can be consistently written as an asymptotic part (equal to the lapse and shifts of spacelike Minkowski hyperplanes) plus a part which vanishes at spatial infinity. Since spacelike Minkowski hyperplanes are described in phase space by 10 configuration variables (an origin plus an orthonormal tetrad) plus the conjugate momenta (see the parametrized Minkowski theories), Dirac adds these 20 variables to the ADM phase space, but then he also adds 10 first class constraints to the Dirac Hamiltonian (so that the 10 configurational variables are gauge variables). These constraints determine the 10 extra momenta in terms of the 10 weak Poincaré charges.

The satisfaction of all the requirements on the boundary conditions of the 16 fields and of the gauge transformations, in particular the absence of supertranslations, leads to the following results. The Hamiltonian formulation of both metric and tetrad gravity is well posed for the class of Christodoulou-Klainermann spacetimes [22], which are near Minkowski spacetime in a norm sense and avoid the singularity theorems not admitting a conformal completion, but which contain asymptotic gravitational radiation at null infinity (even if with a weaker peeling of the Weyl tensor). The allowed 3+1 splittings for these spacetimes have all the leaves Σ_τ approaching, in a direction-independent way, those special Minkowski hyperplanes asymptotically orthogonal to the weak ADM Poincaré 4-momentum. These asymptotic spacelike hyperplanes are the analogue of the Wigner hyperplanes of parametrized Minkowski theories, and, when matter is present, allow to deparametrize tetrad gravity so to obtain the description of the same matter in the rest-frame instant form on Wigner hyperplanes in Minkowski spacetime when the Newton constant is switched off. Therefore, this Hamiltonian treatment of the Christodoulou-Klainermann spacetimes is the rest-frame instant form of general relativity; like in parametrized Minkowski theories, there is a decoupled canonical non-covariant "external" center of mass (a point particle clock) now located near spatial infinity, while all the physical degrees of freedom are relative variables (a weak form of Mach principle). These asymptotic hyperplanes are privileged observers dynamically selected by the given configuration of the gravitational field (they replace the "fixed stars") and not a priori given like in bimetric theories or in theories with a background. It can be shown that given an asymptotic tetrad determined by the ADM 4-momentum, this tetrad can be transported in each point of Σ_τ by using the Frauendiener equations[23] with the Sen connection (replacing the Sen-Witten equations [24] for spinors in the case of triads and tetrads), so determining a dynamically selected privileged timelike congruence

of observers. These spacelike hypersurfaces Σ_τ can be called Wigner-Sen-Witten (WSW) hypersurfaces.

Given this framework, it is possible to solve the rotation and supermomentum constraints and to find parametrizations of the cotriads in terms of:
i) the 3 gauge rotation angles;
ii) the 3 gauge parameters associated with the pseudodiffeomorphisms, namely with the choice of the 3-coordinates;
iii) the conformal factor of the 3-metric;
iv) the two physical degrees of freedom describing the gravitational field.
Each choice of the 3-coordinates on Σ_τ turns out to be equivalent to the choice of a particular parametrization of the cotriad (see Refs.[25] for previous attempts). In this way 13 of the 14 first class constraints are under control and we can do a Shanmugadhasan canonical transformation adapted to these 13 constraints.

We have till now studied the most natural choice of 3-coordinates, which corresponds to the 3-orthogonal gauges in which the 3-metric is diagonal (they are the nearest ones to the standards of the non-inertial physical laboratories on the earth). The 3 rotation angles and the 3 boost parameters are put equal to zero. As a last gauge fixing we put equal to zero the momentum conjugate to the conformal factor of the 3-metric, which, in turn, must be determined as solution of the Lichnerowicz equation in this gauge. By going to Dirac brackets with respect to the 14 constraints and the 14 gauge-fixings, we remain with two pairs of canonical variables , describing the Hamiltonian physical degrees of freedom or Dirac's observables of the gravitational field in this completely fixed 3-orthogonakl gauge (complete breaking of general covariance). The physical Hamiltonian for the evolution in the mathematical time parametrizing the WSW hypersurfaces Σ_τ of the foliation is the weak (volume form) ADM energy [26]: it depends only on the Dirac's observables, even if part of the dependence is through the conformal factor of the 3-metric, whose form is unknown since noone is able to solve the Lichnerowicz equation. The physical times (atomic clocks, ephemeris times,..) have to be locally correlated to this mathematical time.

Also the Komar-Bergmann individuating fields [27], needed for a physical identification of the points of the spacetime M^4 (due to general covariance the mathematical points of M^4 have no physical meaning in absence of a background; see Einstein's hole argument), may be re-expressed in terms of Dirac's observables.

Finally the Poincaré Casimirs associated with the asymptotic weak Poincaré charges allow to define the Møller radius (and a possible ultraviolet cutoff in a future attempt to make a quantization of completely gauge-fixed tetrad gravity) also for the gravitational field.

The main tasks for the future are:

A) Make the canonical quantization of scalar electrodynamics in the rest-frame instant form on the Wigner hyperplanes, which should lead to a particular realization of Tomonaga-Schwinger quantum field theory, avoiding the no-go theorems of Refs.[28]. The Møller radius should be used as a physical ultraviolet cutoff for

the point splitting technique and the results of Refs.[29] about the infrared dressing of asymptotic states in S matrix theory should help to avoid the 'infraparticle' problem[30].

B) Study the linearization of tetrad gravity in the 3-orthogonal gauge to reformulate the theory of gravitational waves in this gauge.

C) Study the N-body problem in tetrad gravity at the lowest order in the Newton constant (action-at-a-distance plus linearized tetrad gravity). See Ref.[1] for preliminary results on the action at a distance hidden in Einstein's theory at the lowest order in the Newton constant, which agree with the old results of Ref.[31].

D) Study the perfect fluids both in the rest-frame instant form in Minkowski spacetime [32] and in tetrad gravity.

E) Make the Hamiltonian reformulation of the Newman-Penrose formalism[33] by using Hamiltonian null tetrads and study its connection with the 2+2 decompositions of M^4[34].

F) Begin the study of the standard model of elementary particles coupled to tetrad gravity starting from the Einstein-Maxwell system.

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